

# EE 230

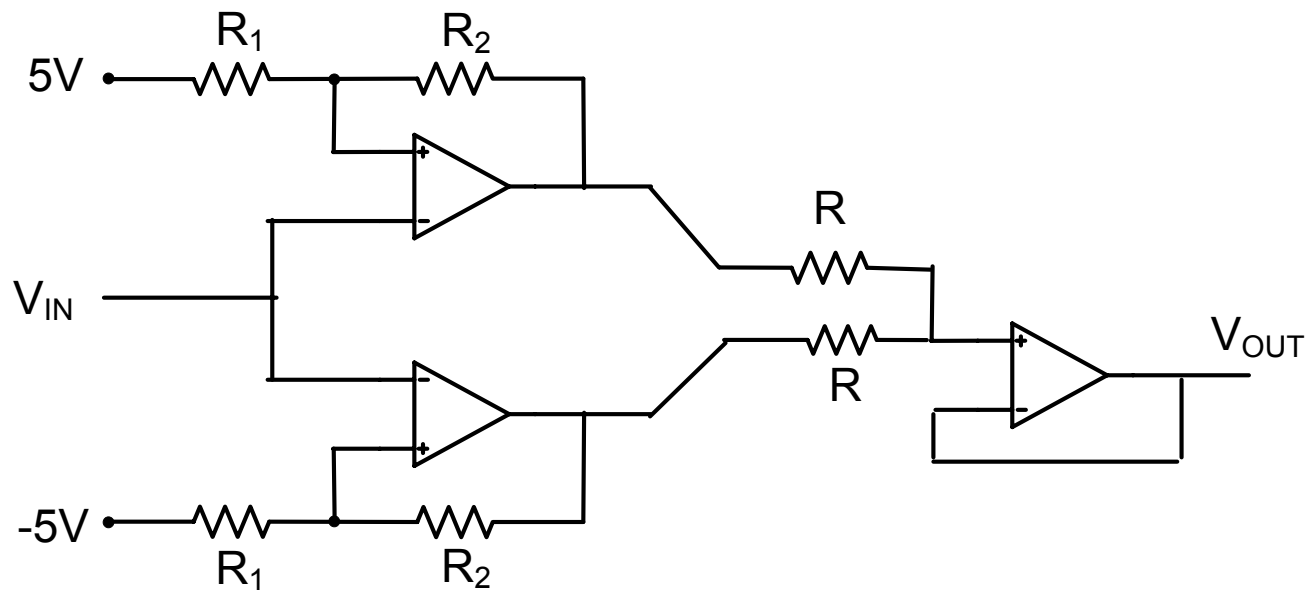
## Lecture 23

Nonlinear Op Amp Applications

- waveform generators

# Quiz 16

Obtain an expression for and plot the transfer characteristics of the following circuit. Assume  $R_1=2K$ ,  $R_2=8K$ ,  $R=10K$ ,  $V_{DD}=+15V$ ,  $V_{SS}=-15V$



And the number is ?

1

3

8

5

4

2

6

9

7

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1

3

8

5

4

2

3

6

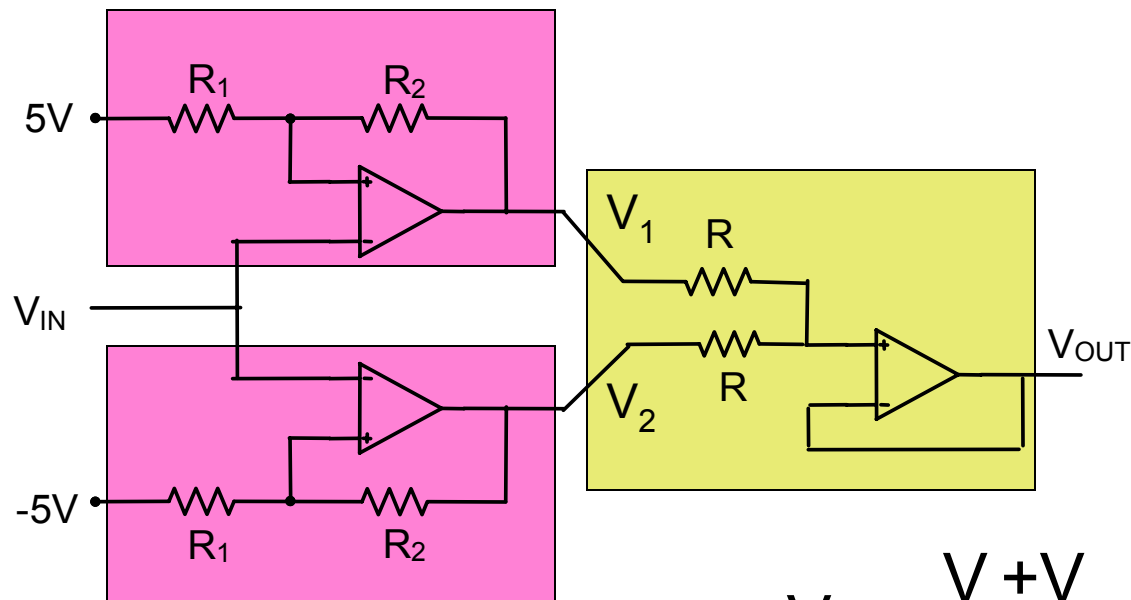
9

7

# Quiz 16

Solution:

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$$V_{OUT} = \frac{V_1 + V_2}{2}$$

$$V_{HYH} = \theta V_{SATL} + (1-\theta) V_R$$

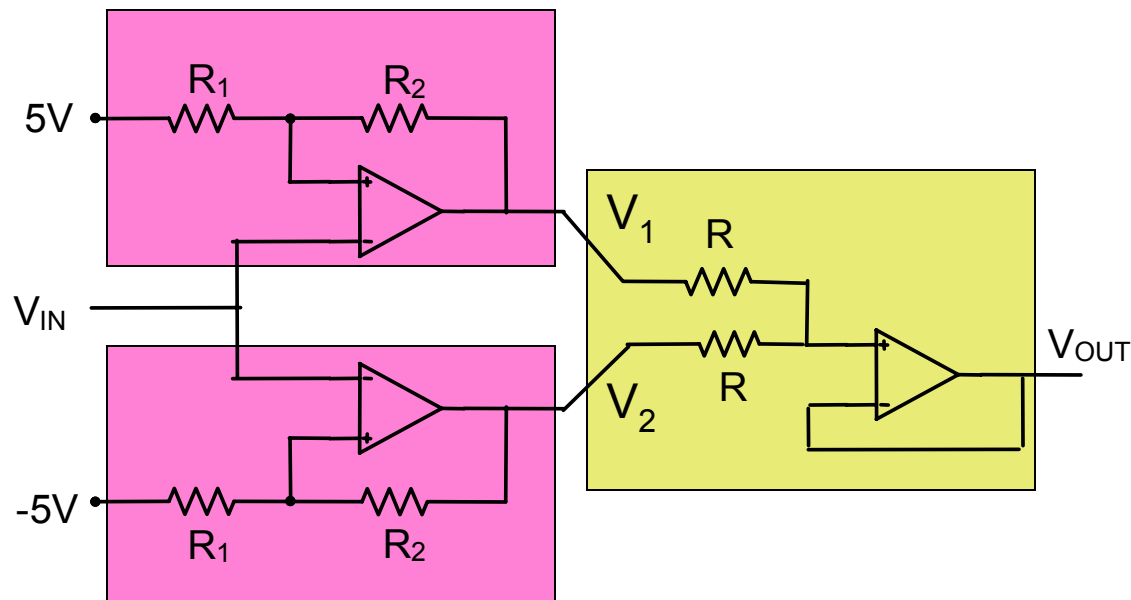
$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R$$

$$\theta = \frac{R_1}{R_1 + R_2}$$

# Quiz 16

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$$\theta = \frac{R_1}{R_1 + R_2} = 0.2$$

$$V_{HYH} = \theta V_{SATL} + (1-\theta) V_R = 3V + 4V = 7V$$

$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R = -3V + 4V = 1V$$

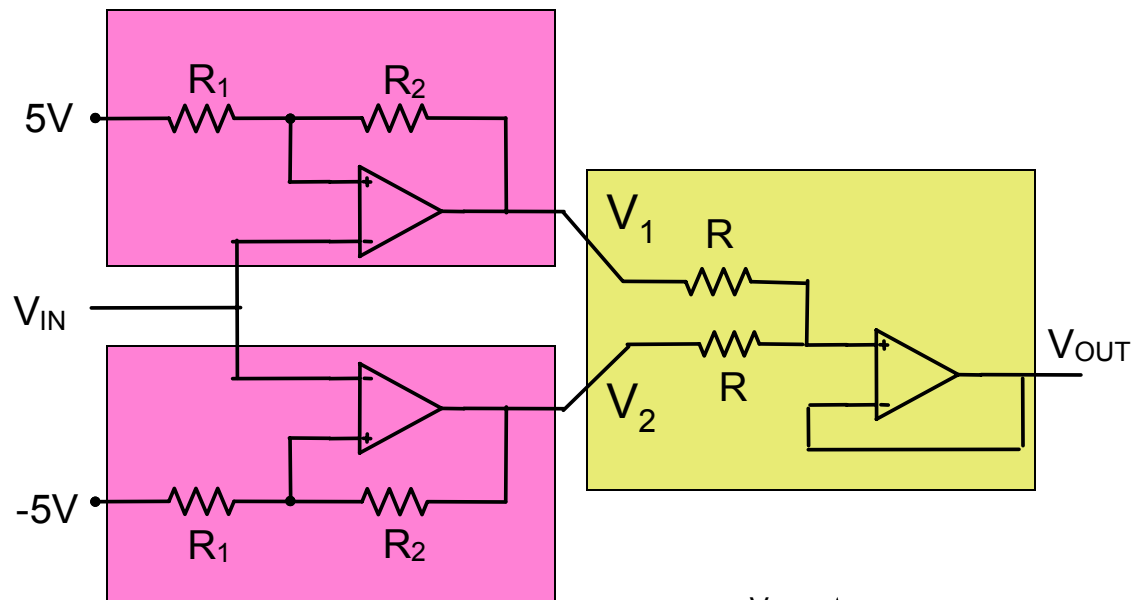
$$V_{HYH} = \theta V_{SATL} + (1-\theta) V_R = 3V - 4V = -1V$$

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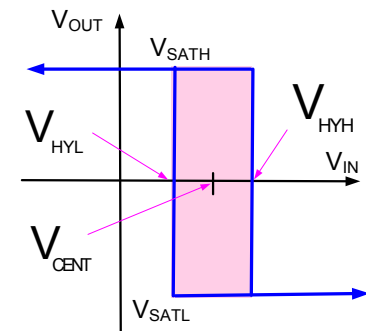
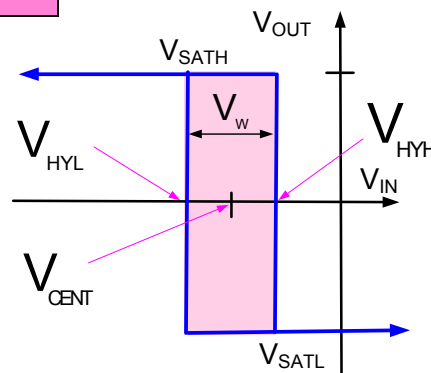


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# Quiz 16

Solution:

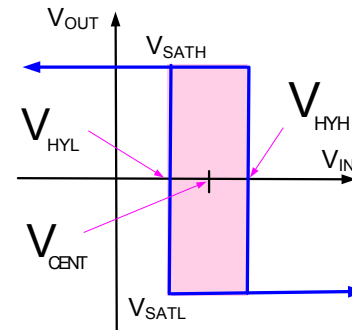
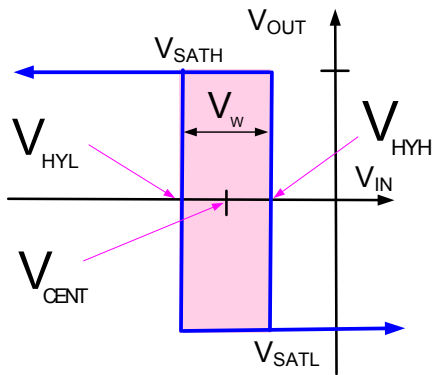
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$$V_{HYH} = \theta V_{SATH} + (1-\theta) V_R = 3V - 4V = -1V$$

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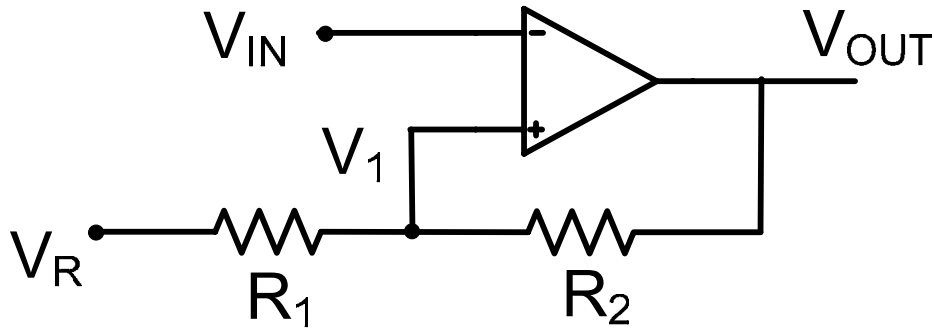
$$V_{HYL} = \theta V_{SATL} + (1-\theta) V_R = -3V - 4V = -7V$$

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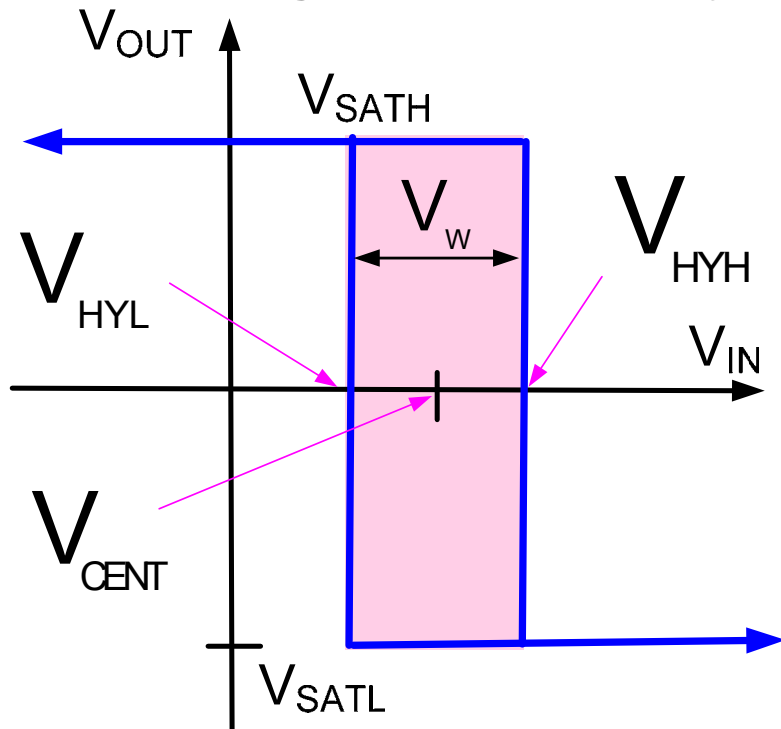
Review from Last Time:



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_W = \theta (V_{SATL} - V_{SATH})$$

**Shifted Inverting Comparator with Hysteresis**



$$V_{CENT} = \left( \frac{\theta (V_{SATH} + V_{SATL})}{2} \right) + (1 - \theta) V_R$$

If  $V_{SATH} = V_{DD}$ ,  $V_{SATL} = V_{SS} = -V_{DD}$

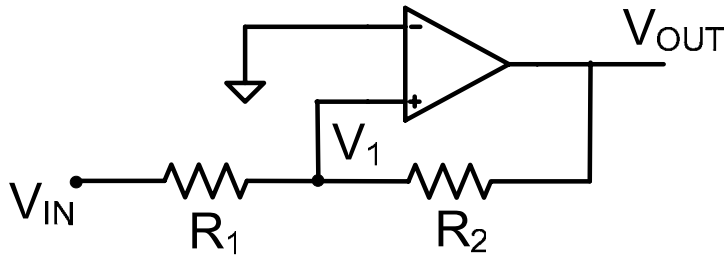
$$V_W = 2\theta V_{DD}$$

$$V_{CENT} = (1 - \theta) V_R$$

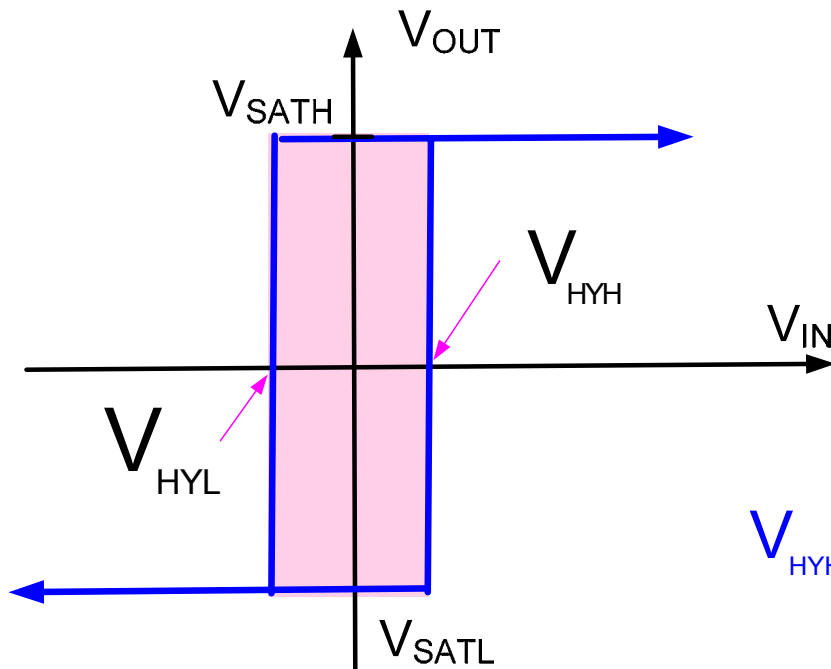
**Shift can be to left or right depending upon sign of  $V_R$**

Review from Last Time:

# Inversion of Hysteresis Loop



**Noninverting Comparator with Hysteresis**



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \quad V_{SATL} \cong V_{SS}$$

$$V_{HYH} = \frac{\theta}{\theta - 1} V_{SATL}$$

$$V_{HYL} = \frac{\theta}{\theta - 1} V_{SATH}$$

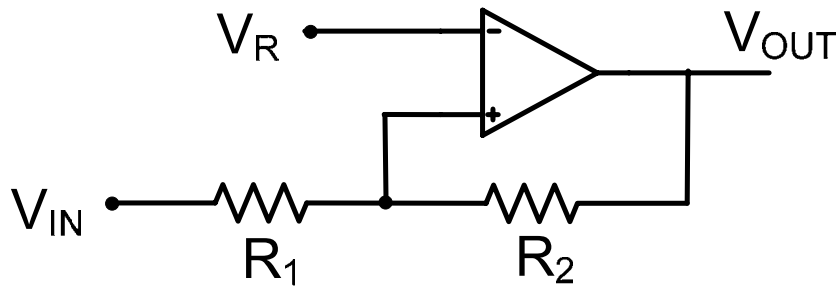
If  $V_{SATH} = V_{DD}$ ,  $V_{SATL} = V_{SS} = -V_{DD}$

$$V_{HYH} = \frac{\theta}{1 - \theta} V_{DD}$$

$$V_{HYL} = \frac{-\theta}{1 - \theta} V_{DD}$$

Review from Last Time:

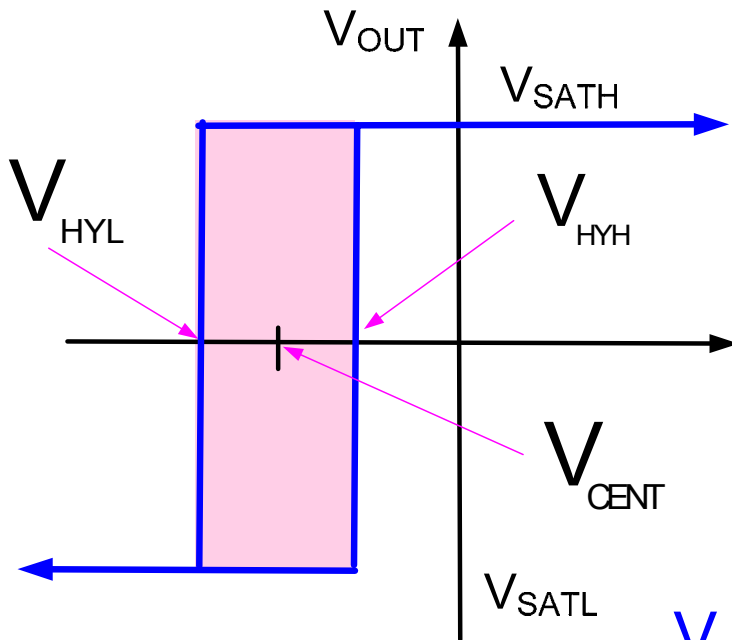
# Shifted Inverted Hysteresis Loop



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \quad V_{SATL} \cong V_{SS}$$

## Shifted Noninverting Comparator with Hysteresis



$$V_{HYH} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} V_{SATL}$$

$$V_{HYL} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} V_{SATH}$$

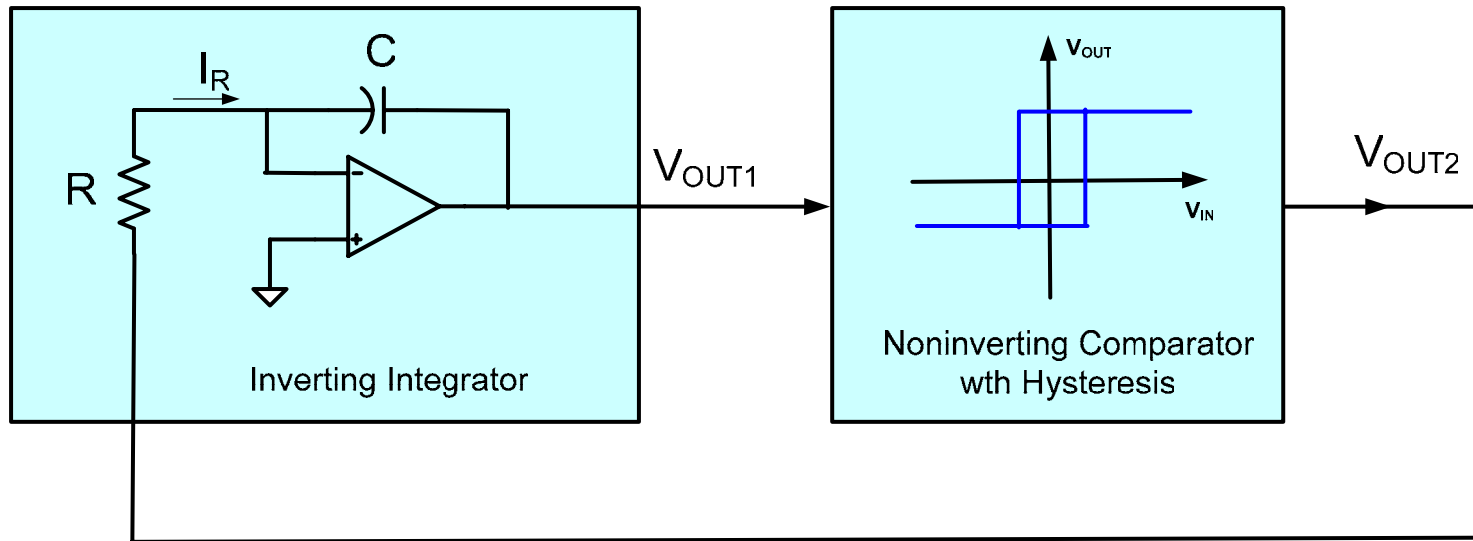
$$V_{CENT} = \frac{V_R}{1-\theta} + \frac{\theta}{\theta-1} \left( \frac{V_{SATH} - V_{SATL}}{2} \right)$$

If  $V_{SATH} = V_{DD}$ ,  $V_{SATL} = V_{SS} = -V_{DD}$

$$V_{HYH} = \frac{V_R}{1-\theta} + \frac{\theta}{1-\theta} V_{DD}$$

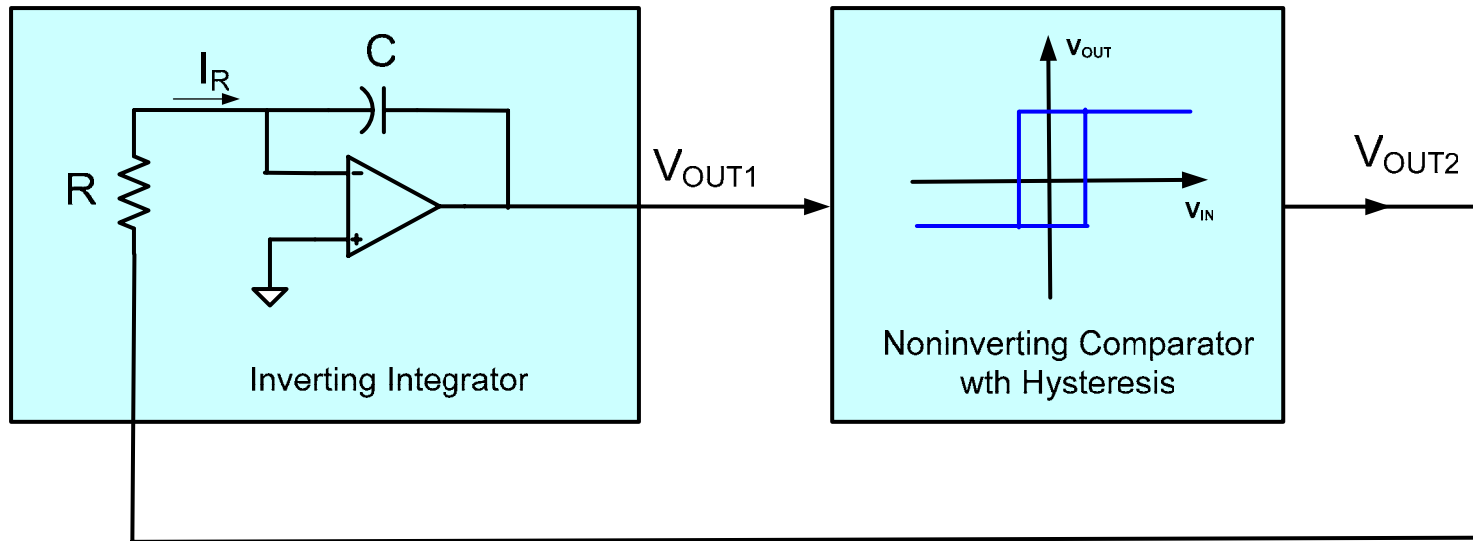
$$V_{HYL} = \frac{V_R}{1-\theta} + \frac{-\theta}{1-\theta} V_{DD}$$

# Waveform Generator with Linear Triangle Waveform



Goal: Determine how this circuit operates, the output waveforms, and the frequency of the output

# Waveform Generator with Linear Triangle Waveform



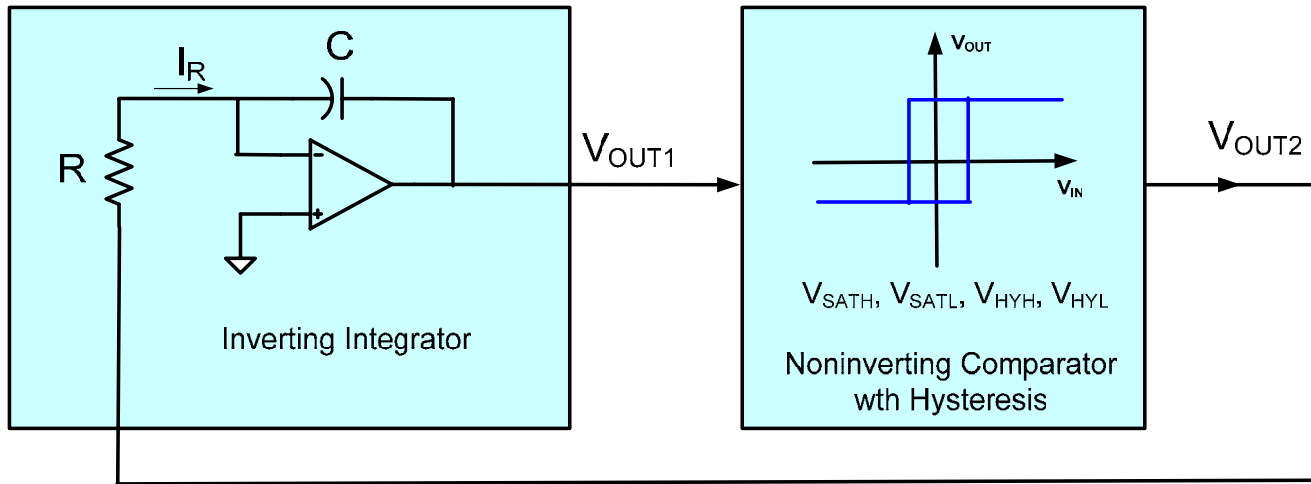
Since the comparator will be in one of two states, the current in the resistor will be constant when  $V_{OUT2} = V_{SATH}$  and will be constant when  $V_{OUT2} = V_{SATL}$

Analysis strategy: Guess state of the  $V_{OUT2}$ , solve circuit, and show where valid

when  $V_{OUT2} = V_{SATH}$ ,  $I_R$  will be positive and  $V_{OUT1}$  will be decreasing linearly

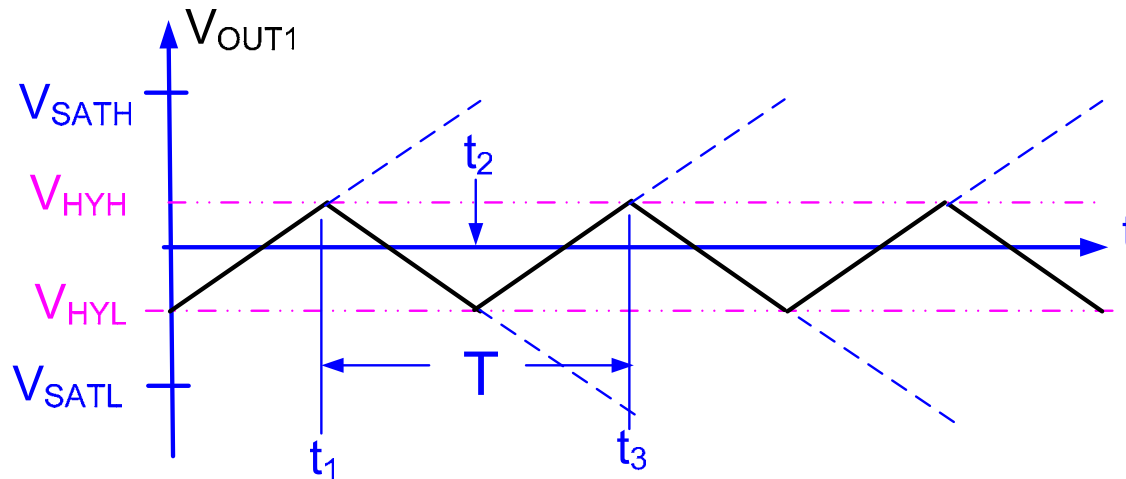
when  $V_{OUT2} = V_{SATL}$ ,  $I_R$  will be negative and  $V_{OUT1}$  will be increasing linearly

# Waveform Generator with Linear Triangle Waveform



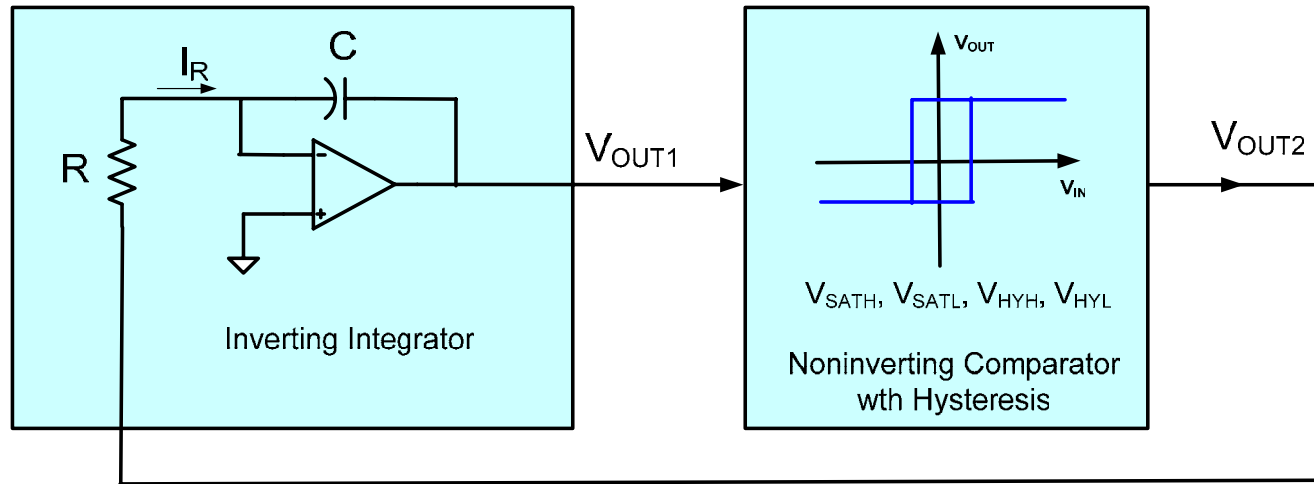
$$V_{SATH} \approx V_{DD}$$

$$V_{SATL} \approx V_{SS}$$



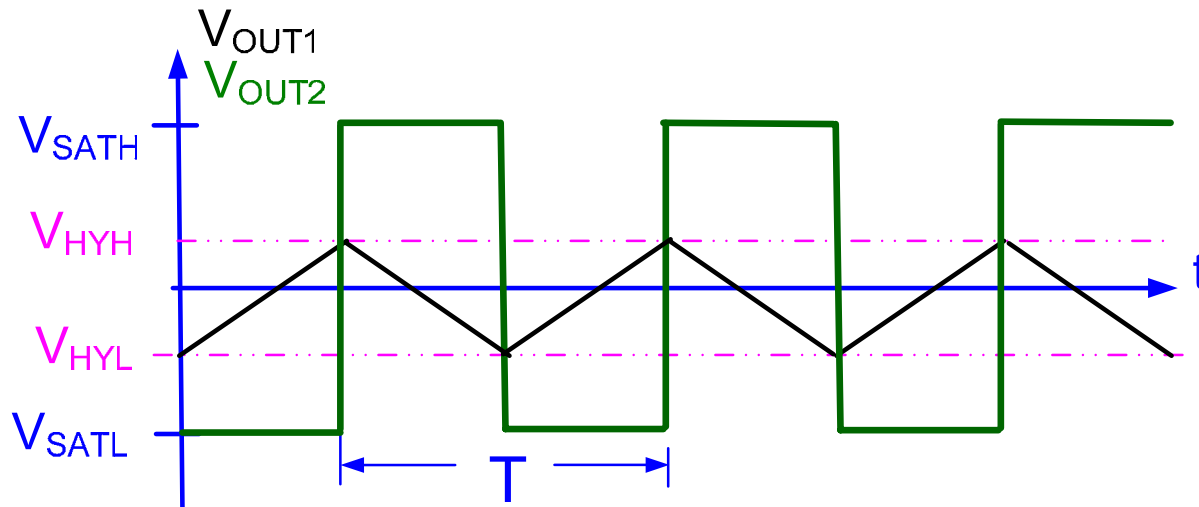
Observe  $T = t_3 - t_1 = (t_2 - t_1) + (t_3 - t_2)$

# Waveform Generator with Linear Triangle Waveform

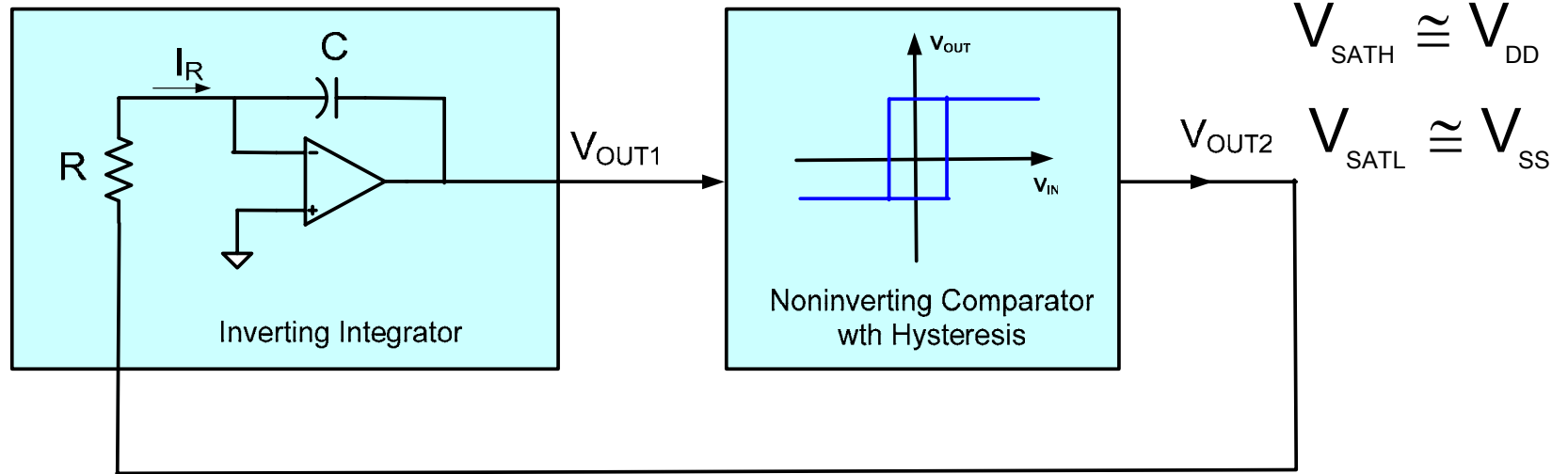


$$V_{SATH} \approx V_{DD}$$

$$V_{SATL} \approx V_{SS}$$



# Waveform Generator with Linear Triangle Waveform



Guess  $V_{OUT2} = V_{SATH}$  **will obtain  $t_2 - t_1$**

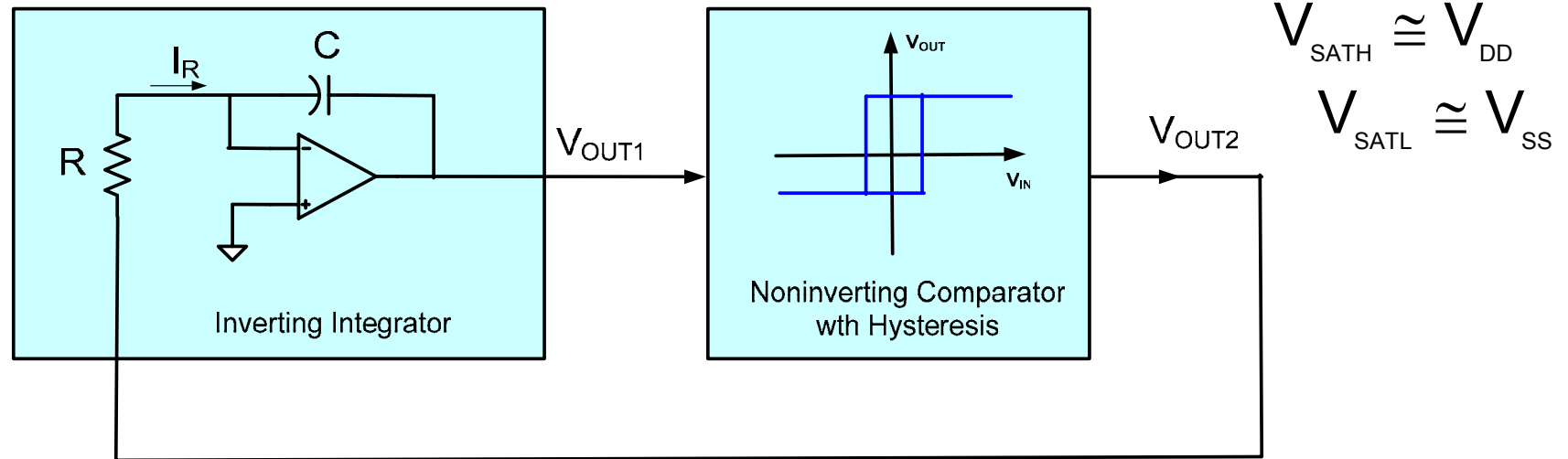
$$V_{OUT1} = -\frac{1}{RC} \int_{t_1}^t V_{SATH} d\tau + V_{OUT1}(t_1)$$

$$V_{OUT1}(t_1) = V_{HYH}$$

valid for  $t_1 < t < t_2$



# Waveform Generator with Linear Triangle Waveform



Guess  $V_{OUT2} = V_{SATH}$  valid for  $t_1 < t < t_2$

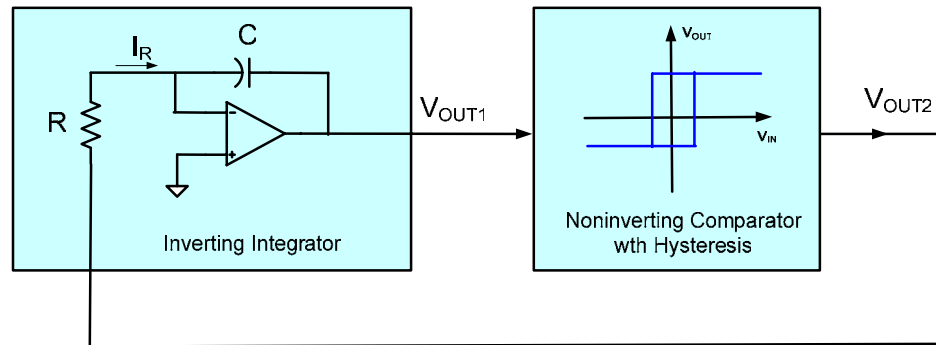
$$V_{OUT1} = -\frac{1}{RC} \int_{t_1}^t V_{SATH} d\tau + V_{OUT1}(t_1) \quad V_{OUT1}(t_1) = V_{HYH}$$

at  $t=t_2$ ,  $V_{OUT1}$  will become  $V_{SATL}$

Substituting into integral expression for  $V_{OUT1}$  we obtain

$$V_{HYL} = -\frac{1}{RC} \int_{t_1}^t V_{SATH} d\tau + V_{HYH}$$

# Waveform Generator with Linear Triangle Waveform



$$V_{SATH} \cong V_{DD}$$

$$V_{SATL} \cong V_{SS}$$

Guess  $V_{OUT2} = V_{SATH}$  valid for  $t_1 < t < t_2$

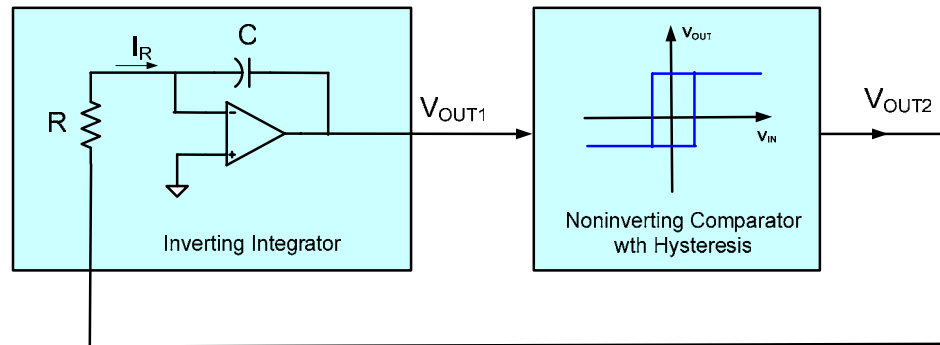
$$V_{HYL} = -\frac{1}{RC} \int_{t_1}^t V_{SATH} d\tau + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC} V_{SATH} \int_{t_1}^{t_2} 1 d\tau + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC} V_{SATH} (\tau|_{t_1}^{t_2}) + V_{HYH}$$

$$V_{HYL} = -\frac{1}{RC} V_{SATH} (t_2 - t_1) + V_{HYH}$$

# Waveform Generator with Linear Triangle Waveform



$$V_{\text{SATH}} \cong V_{\text{DD}}$$

$$V_{\text{SATL}} \cong V_{\text{SS}}$$

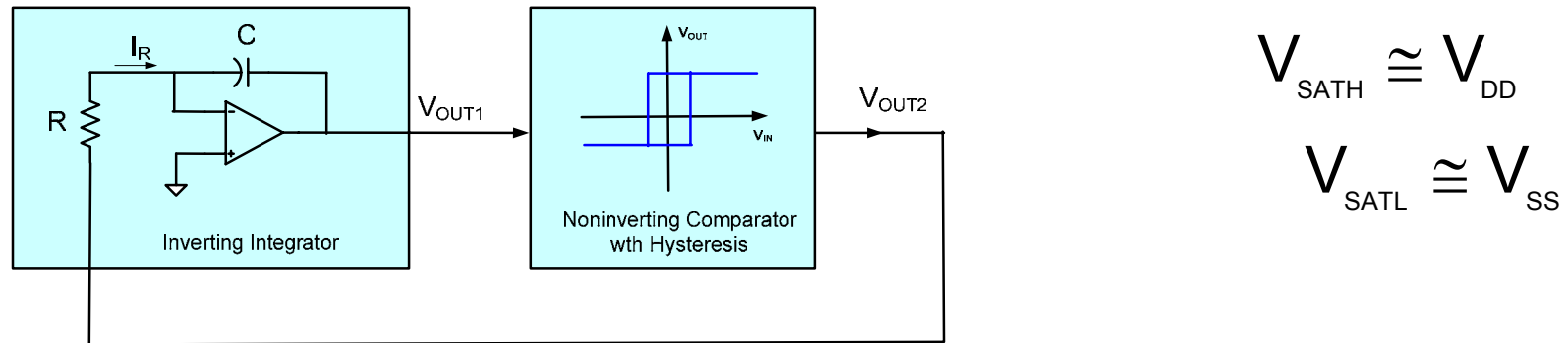
Guess  $V_{\text{OUT2}} = V_{\text{SATH}}$

valid for  $t_1 < t < t_2$

$$V_{\text{HYL}} = -\frac{1}{RC} V_{\text{SATH}} (t_2 - t_1) + V_{\text{HYH}}$$

$$t_2 - t_1 = RC \frac{(V_{\text{HYH}} - V_{\text{HYL}})}{V_{\text{SATH}}}$$

# Waveform Generator with Linear Triangle Waveform



Guess  $V_{OUT2} = V_{SATL}$  will obtain  $t_3 - t_2$  valid for  $t_2 < t < t_3$

Following the same approach observe

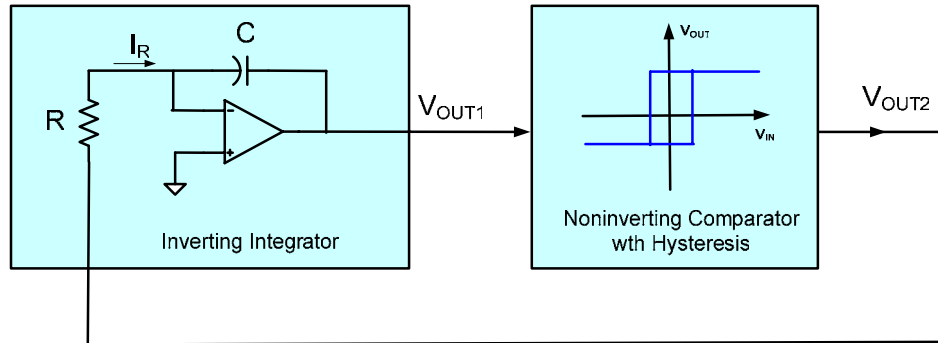
$$V_{OUT1} = -\frac{1}{RC} \int_{t_2}^t V_{SATL} d\tau + V_{OUT1}(t_2)$$

$$V_{OUT1}(t_2) = V_{HYL}$$

It thus follows that

$$V_{HYH} = -\frac{1}{RC} V_{SATL} (t_3 - t_2) + V_{HYL} \quad t_3 - t_2 = RC \frac{(V_{HYL} - V_{HYH})}{V_{SATL}}$$

# Waveform Generator with Linear Triangle Waveform



$$V_{\text{SATH}} \cong V_{\text{DD}}$$

$$V_{\text{SATL}} \cong V_{\text{SS}}$$

$$T = (t_2 - t_1) + (t_3 - t_2)$$

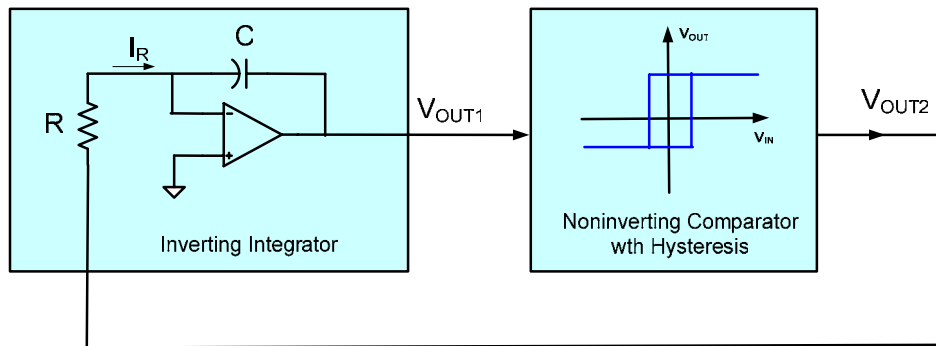
$$t_2 - t_1 = RC \frac{(V_{\text{HYH}} - V_{\text{HYL}})}{V_{\text{SATH}}}$$

$$t_3 - t_2 = RC \frac{(V_{\text{HYL}} - V_{\text{HYH}})}{V_{\text{SATL}}}$$

$$T = RC (V_{\text{HYH}} - V_{\text{HYL}}) \left( \frac{1}{V_{\text{SATH}}} - \frac{1}{V_{\text{SATL}}} \right)$$

$$f = \frac{1}{t} = \frac{1}{RC (V_{\text{HYH}} - V_{\text{HYL}}) (V_{\text{SATL}} - V_{\text{SATH}})}$$

# Waveform Generator with Linear Triangle Waveform



$$f = \frac{1}{RC} \frac{V_{SATL} V_{SATH}}{(V_{HYH} - V_{HYL})(V_{SATL} - V_{SATH})}$$

If we use the noninverting comparator with hysteresis circuit developed previously and if

If  $V_{SATH} = V_{DD}$ ,  $V_{SATL} = V_{SS} = -V_{DD}$   $\theta = \frac{R_1}{R_1 + R_2}$

then

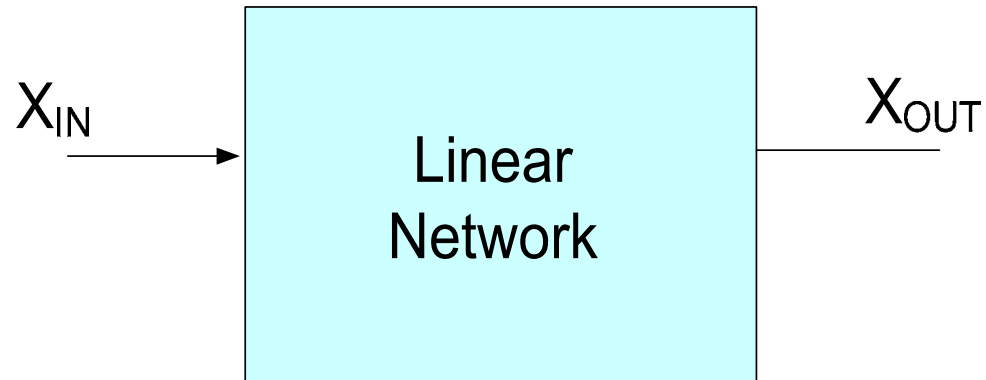
$$V_{HYH} = \frac{\theta}{1-\theta} V_{DD} \quad V_{HYL} = \frac{-\theta}{1-\theta} V_{DD}$$

$$f = \frac{1}{2RC} \frac{1-\theta}{\theta}$$

# Stability and Waveform Generation

- Waveform generators provide an output with no excitation
- Waveform circuits are circuits that, when operated in quiescent linear condition, have one or more poles in the right half-plane
- Will now investigate the pole locations of waveform generators
  - Conditions for oscillation
  - Triangle/Square/Sinusoidal Oscillations

# Poles of a Network



$T(s)$  can be expressed as

$$T(s) = \frac{X_{OUT}(s)}{X_{IN}(s)}$$

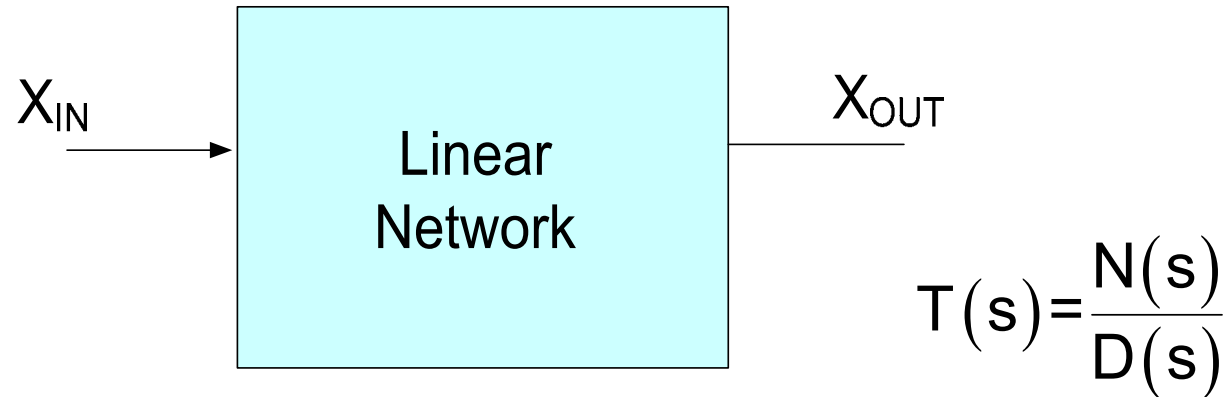
$$T(s) = \frac{N(s)}{D(s)}$$

where  $N(s)$  and  $D(s)$  are polynomials in  $s$

- $D(s)$  is termed the characteristic equation or the characteristic polynomial of the network
- Roots of  $D(s)$  are the poles of the network



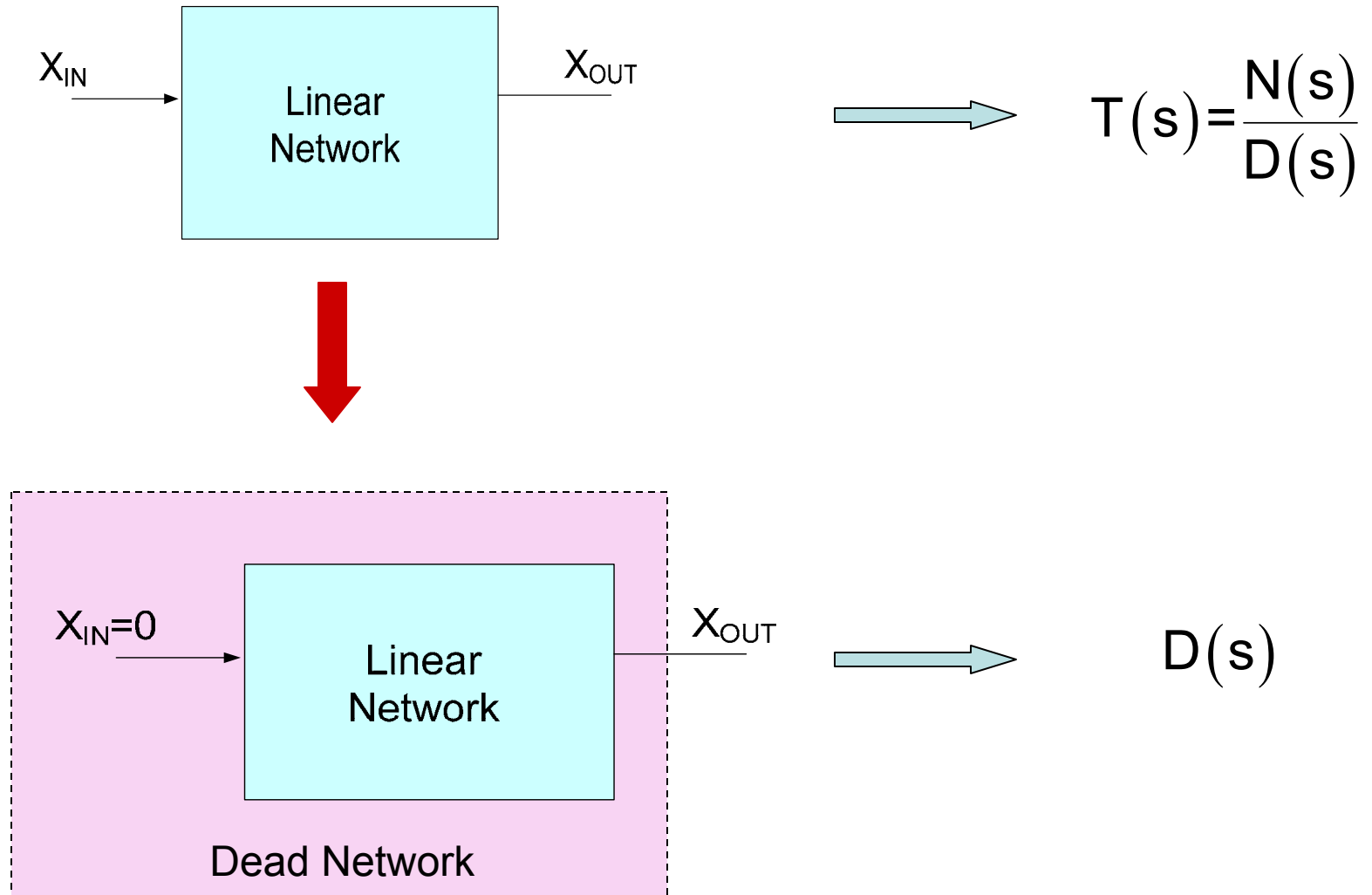
# Poles of a Network



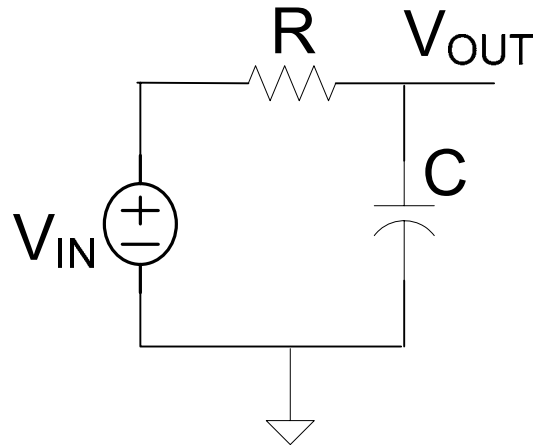
Theorem: The poles of any transfer function of a linear system are independent of where the excitation is applied and where the response is taken provided the dead networks are the same

Equivalently, the characteristic equation,  $D(s)$ , is characteristic of a network (or the corresponding dead network) and is independent of where the excitation is applied and where the response is taken.

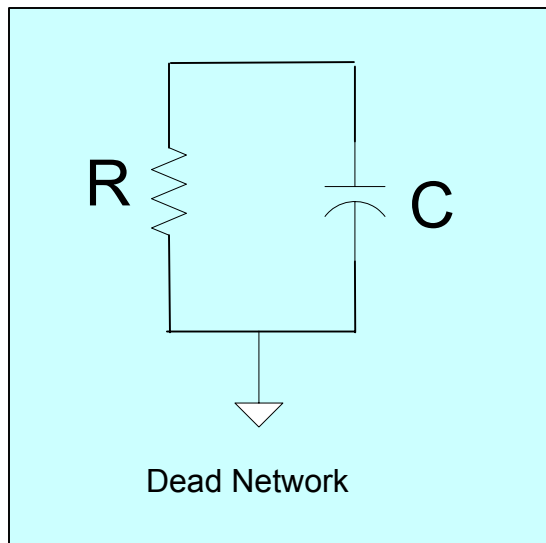
# Poles of a Network



# Poles of Networks – some examples

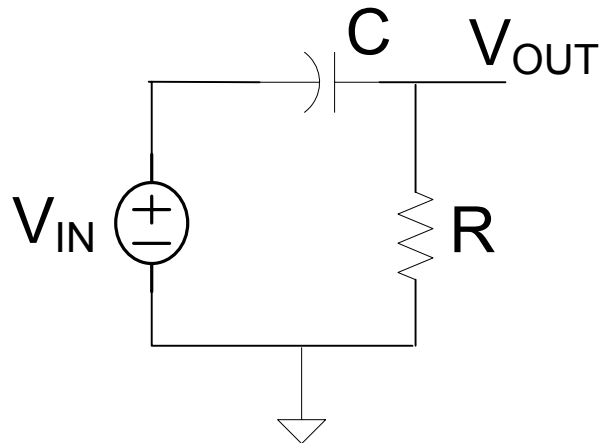


$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1+RCs}$$

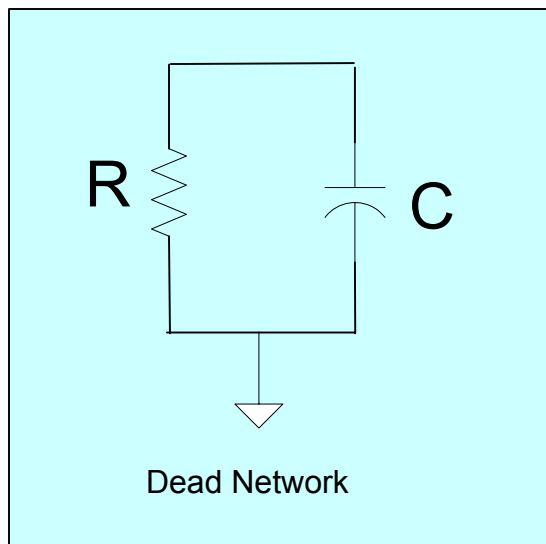


$$D(s) = 1 + RCs$$

# Poles of Networks – some examples

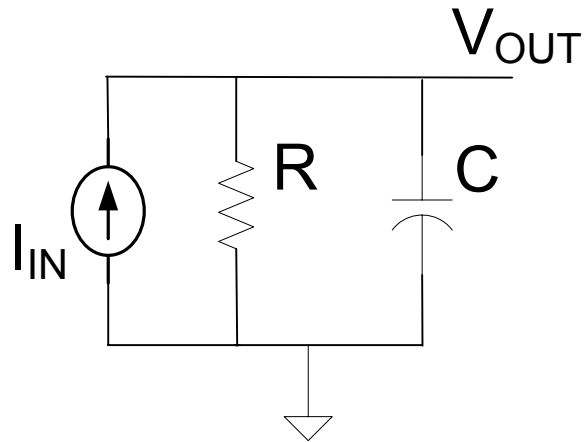


$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{RCs}{1+RCs}$$

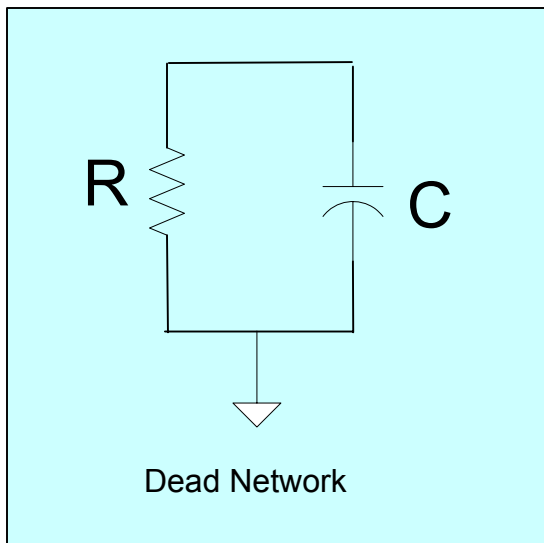


$$D(s) = 1 + RCs$$

# Poles of Networks – some examples

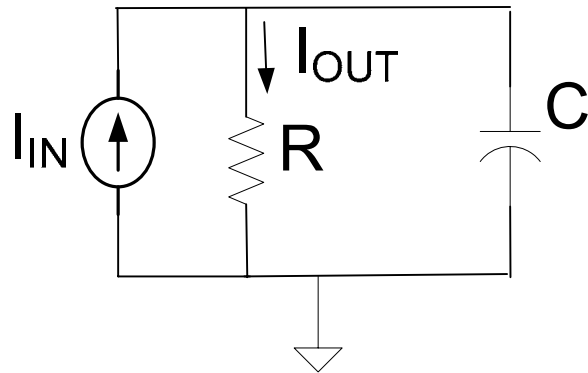


$$T(s) = \frac{V_{OUT}}{I_{IN}} = \frac{R}{1+RCs}$$

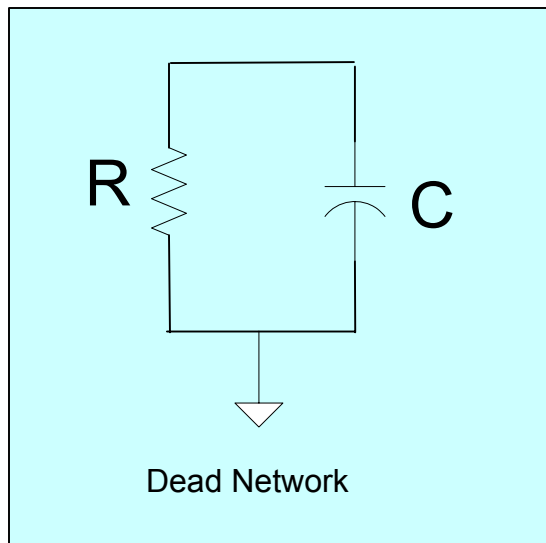


$$D(s) = 1 + RCs$$

# Poles of Networks – some examples

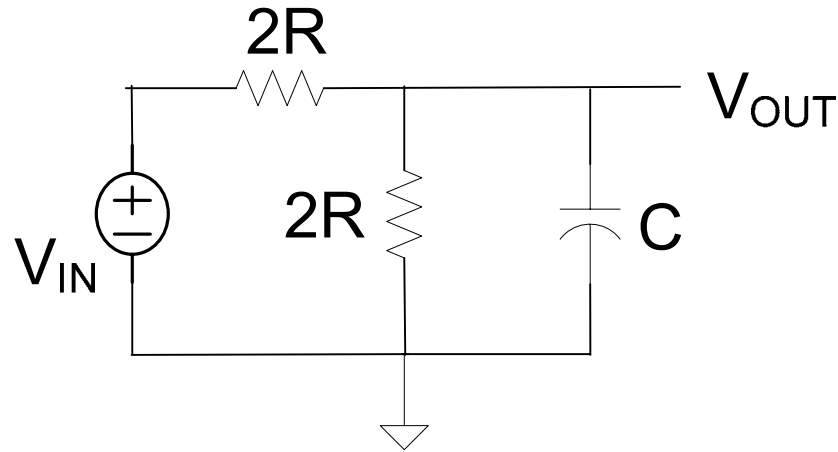


$$T(s) = \frac{I_{OUT}}{I_{IN}} = \frac{1}{1+RCs}$$

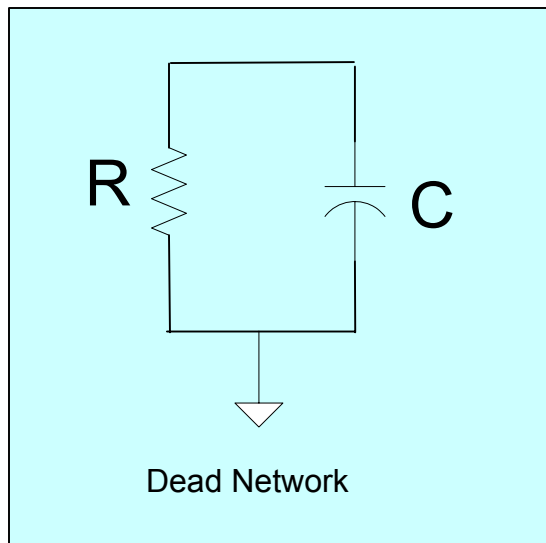


$$D(s) = 1 + RCs$$

# Poles of Networks – some examples

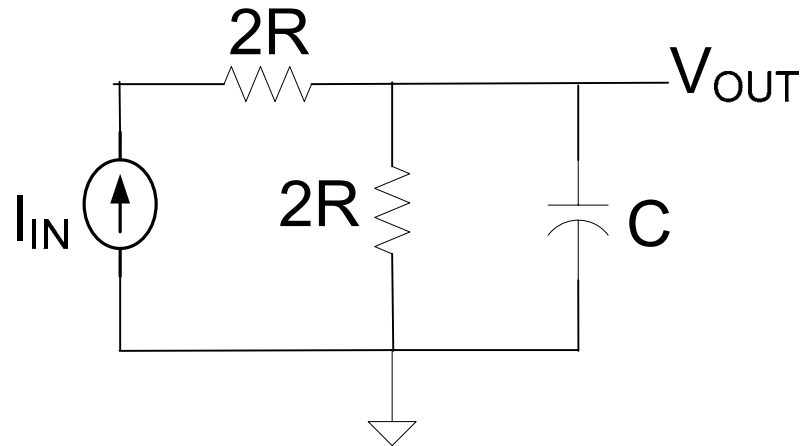


$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + RCs}$$

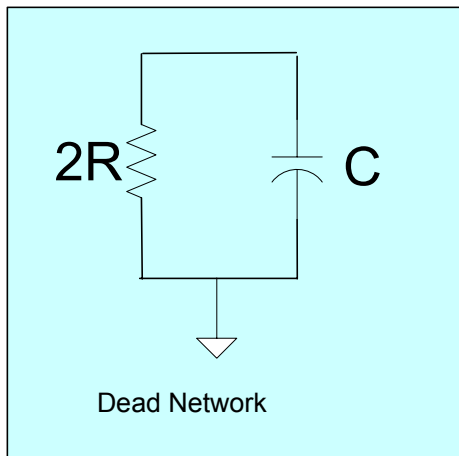


$$D(s) = 1 + RCs$$

# Poles of Networks – some examples



$$T(s) = \frac{V_{OUT}}{I_{IN}} = \frac{2R}{1+2RCs}$$



$$D(s) = 1 + 2RCs$$

Note dead network has changed as has  $D(s)$  and thus the pole